

# Are quantum systems physical objects with physical properties?

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Received 13 September 1999

**Abstract.** Despite its power as the conceptual basis for a huge range of physical phenomena in atomic and subatomic physics, quantum mechanics still suffers from a lack of clarity regarding the physical meaning of its fundamental theoretical concepts, such as those of *quantum state* and of *quantum theoretical quantities or variables*, dealt with by the known mathematical-theoretical rules. These concepts have generally been considered as not giving a *direct description* of physical systems, for they do not correspond to what is usually understood by ‘physical states’ or ‘physical properties’, notably characterized by definite numerical values such as those obtained from measurement. The situation has been tentatively expressed in terms of various ‘interpretations’, conceived not only with regard to the physical meanings of mathematical quantities, but also in connection with philosophical statements about ‘physical reality’. The question of whether or not quantum theoretical quantities describe definite physical systems existing in nature has therefore generally been considered as beyond the possibilities of physics, because of the definitions that are commonly taken for ‘physical state’ and ‘physical quantity’. We sketch the main ideas on this problem and propose a possible way out of the puzzle, in terms of an extension of the meaning given to the concepts of the *physical state* and *physical quantity* of a system, which would allow one, without any theoretical change in quantum mechanics, to speak consistently of *real quantum systems* as having definite *physical properties*.

## 1. Introduction. The physical interpretation of quantum mechanics and the statue with feet of clay

There is no need to recall the power of quantum theory (the theory of quantum physics) in imparting knowledge of atomic and subatomic matter, from the constitution of the bodies of our environment as molecular associations of atoms up to the deep structure of matter in general: that of atomic nuclei, of elementary particles actually or ‘virtually’ contained in the latter, and also that of cosmic objects and of the Universe considered in its ‘elementary stage’, providing the germs of its future states, as manifested in the primordial phases of cosmology. Clearly, this powerful theory still remains incomplete or unachieved with regard to such an unusually ambitious project that notwithstanding is generally considered legitimate and within the reach of human thought. Further endeavours in this direction will be based on the present quantum theory as a well established body of knowledge and we shall stay, in what follows, within the scope of its present characteristics, these being taken for granted.

There is, however, a paradox in the situation of quantum theory, which is related to the so-called ‘foundational problems of quantum mechanics’. The paradox is that this theory, so powerful in describing matter, is unsure of the exact meaning of its statements. Its description of ‘elements of the material world’ or ‘physical states’ (atoms, elementary particles, interaction

fields, etc) makes use of ‘state functions’ and ‘quantities’ (or ‘variables’), more specifically ‘state functions of quantum systems’ and ‘quantum variables’. These have very precise mathematical expressions and are submitted to definite rules of utilization as dynamical variables, but their physical meaning is far from being direct and remains today a disputed question among physicists.

It is true that *quantum theory* is by now much wider than *quantum mechanics* in the restricted sense, for it not only includes further theoretical models for atomic and nuclear properties, but has been extended, from a more fundamental point of view, into *quantum field theory*, from quantum electrodynamics (QED)<sup>†</sup> to electroweak and chromodynamic gauge field theories (see, for example, Leite Lopes 1981, Bimbot and Paty 1996). In the quantum-field theoretical sense the concept of ‘state function’ is not as simple as in genuine quantum mechanics, where it is a solution of the Schrödinger or Dirac equations expressing the laws of the physical system under consideration. Instead of being merely, as in the latter, a *function* in the usual sense, taking numerical values with the variables, a state function in quantum field theory is an *operator*, like other quantum variables in general. The problem of its *physical meaning* or *content* in relation to a *physical system* or *phenomenon* is therefore essentially the same as that of quantum mechanics since its beginnings. One may say, in this respect, that notwithstanding the sophisticated further theoretical developments of quantum physics, *quantum mechanics* continues to be the basic *frame of thought* for the exploration of all kinds of quantum phenomena. The success obtained has, in effect, confirmed its heuristic power and considerably enlarged its basis.

The paradox we alluded to can now be formulated in the following terms: quantum physics can describe and predict an incredibly large number and variety of properties that are ultimately referred to as ‘objects of the world’, and yet at the same time it cannot legitimately claim that its theoretical conceptual entities (*state functions* and *variables* of physical systems) fully represent or describe something that could be consistently called ‘physical objects’. For *physical objects* must have *definite physical properties* which can unambiguously be ascribed to them, and this is not the case for quantum systems whose *properties* are, in general, only *contextual*, depending on the *type of observation* one decides to perform on them, and also, as it is commonly said, on their being observed or not.

It has been a general opinion up to now that quantum physics as it stands has not provided a satisfactory way out of such a paradox, which would make it a giant statue with feet of clay<sup>‡</sup> (‘un colosse aux pieds d’argile’). For some people, this was an indication of the need for another theoretical description, alternative to standard quantum theory, in which physical states would be given direct theoretical expression and the relevant dynamical variables, or theoretical variables, would directly correspond to physical properties. For others, the demand for quantum states and variables to describe definite physical systems actually existing (or ‘real’ systems) is beyond the capabilities of physics, this being evidence of the need for new conceptions about the nature of human knowledge. Both positions leave us fundamentally unsatisfied regarding the aim and nature of physics, and we are forced to examine the meaning of such basic concepts of quantum theory as *state function* and *quantum variable* more deeply with regard to physical phenomena and systems.

We shall see that of the various possible readings of these notions according to the various interpretations, there is one which might well deserve to be favoured from the point of view of intelligibility, supported by familiarization with and practice of the ‘quantum (theoretical) tool’ as well as by recent experimental developments on the production and analysis of *simple specific quantum phenomena*. Besides hypothetical alternative reformulations and ‘orthodox’ interpretations of quantum theory, there seems to be room for a direct and consistent physical interpretation in objective terms (those of a theory that describes physical objects having physical properties), provided one extends the physical meaning of concepts such

<sup>†</sup> On quantum electrodynamics, see for instance, Schwinger (1958) (technical fundamental papers), Feynman (1985) (popular exposition); for its history, see Schweber (1994). See also Leite Lopes and Escoubès (1995).

<sup>‡</sup> From the book of Daniel, ch 2, v 33.

as *state function* and *variable* from mere numerically valued quantities to more complex mathematical forms.

## 2. States, variables and physical meaning. The quantum algorithm

The kind of theory quantum mechanics is, and what it is aimed at, in descriptive terms, is a well known story, and we shall only recall its main lines insofar as they enable us see how it came about that *states* and *variables* have become problematic when considered in the quantum domain, although they seem deeply rooted in ‘what is naturally given to be treated by physics’ (if we commit ourselves to use this ‘puritan’ periphrasis, in order to avoid speaking crudely of ‘the physical reality’). The quantum domain in physics has revealed itself step by step since the very beginning of the 20th century on an essentially ‘phenomenological’ basis, before being expressed theoretically by quantum mechanics.

Phenomena having their source in the radiative and atomic properties of matter could not be accounted for by the then available classical physical theories (electromagnetism, thermodynamics and statistical mechanics) and needed the development of new conceptualized properties that somewhat distorted the usual ones. Such were the *discontinuity in energy* of radiation and of atoms (expressed by means of the fundamental Planck constant<sup>†</sup>), as manifested in the black-body radiation law, in the photoelectric effect, in specific heats and in the atomic-level structure; in the *wave–particle duality* of light radiation and of matter elements<sup>‡</sup>, showing up particularly in interference phenomena (experiments with ‘double-slit’ diaphragms, crystal diffraction of light, of x-rays, later on of electrons and many other quantum particles, including atoms). *The indistinguishability of identical quantum particles* (or ‘systems’) and their ‘*quantum statistical*’ behaviour (‘Bose–Einstein’ for integer-spin particles and ‘Fermi–Dirac’ for half-integer ones) are also remarkable non-classical features of the description of phenomena: the first appeared to be the deep root of Planck’s radiation law and the second explained Pauli’s exclusion principle, responsible for the inner electronic structure of atoms and the periodic classification of the chemical elements. To these must be added *the probabilistic character of the intensity distributions* in radioactive decay and in atomic transition processes, which turned out to be much more fundamental than was first imagined, and as an originary property.

Though formulated in a somewhat theoretical fashion, these *statements about the proper characteristics of quantum phenomena and systems* were the most direct expressions possible for factual situations met within this field. As Einstein, one of the most perspicacious pioneers in establishing quantum physics, used to say, these stated ‘factual’ or ‘empirically given’ properties were compelling in that they gave evidence for a radical break with the previous classical theories of matter and radiation, and were at the same time both imperative and unavoidable conditions that any future theory of the phenomena would have to integrate. This integration would have been performed, it was generally hoped, when one could refer to some fundamental physical principles structuring a theory that would be intelligible in terms of the description of *physical systems* and of the *properties* attached to them, as had always been the case in physics.

Quantum mechanics is the physical theory that has been built up from those ‘conceptualized factual’ features as a ‘closed’, self-consistent theoretical structure. It revealed itself as predictive, leading to new statements about phenomena that were verified. Such were, among others, the *Heisenberg relations* between the widths of spectral distributions of ‘conjugate’ quantities<sup>§</sup>, the explanation of the *tunnelling effect* for alpha particles in a potential well due to the probabilistic interpretation of the ‘wave’ or ‘state function’, the calculation of *the levels of the hydrogen atom*, etc.

<sup>†</sup> For example, in the Planck–Einstein relation between energy ( $E$ ) and frequency ( $\nu$ ) for atomic energy exchanges and for radiation, namely  $E = h\nu$ , with Planck’s constant  $h = 6.55 \times 10^{-27}$  erg s<sup>-1</sup>.

<sup>‡</sup> Another relation, between wavelength and momentum, must be added to the preceding one, namely  $\lambda = h/p$ .

<sup>§</sup> For example, between position ( $x$ ) and momentum ( $p$ ):  $\Delta x \cdot \Delta p \geq \frac{1}{2}\hbar$  (with  $\hbar = h/2\pi$ ).

Taken in its developed formulation<sup>†</sup>, quantum mechanics is based, as all other physical–mathematical theories, on a Lagrangian–Hamiltonian formalism yielding an equation for the dynamical variables that is linear and whose solutions determine the possible states of a physical system (for instance, the electronic levels of the hydrogen atom). However, except for this (essential) feature, quantum mechanics differs from all physical theories previously known with respect to the correspondence that relates the theoretical variables to the physical properties of the system. The state of a system is represented by a ‘state function’ or ‘state vector’<sup>‡</sup>, defined in a (mathematical) Hilbert space<sup>§</sup>, solution of the dynamical state equation<sup>||</sup>. The state function obeys the ‘superposition principle’: any linear superposition of the eigenfunctions is also a possible solution of the equation. As to the variables or dynamical variables attached to the system, they are represented mathematically by linear (Hermitian) operators acting on these state functions or vectors. The value of a given variable ( $A$ ) for a state considered (for example, momentum, spin, etc) is the eigenvalue ( $a$ , a number) of its operator ( $A$ ) corresponding to the eigenvector ( $\psi$ ) representing the state (i.e.  $A\psi = a\psi$ ).

Rules for the physical interpretation of those *mathematically defined variables* are also part of the quantum algorithm. They were not formulated as correspondences of *mathematical quantities* with *physical ones*, as in the previous theories of physics, but with *observed physical ones*, notably through measurement processes. The square of the modulus of the state function gives the probability of the system being (or rather ‘being found’) in the corresponding state, which means therefore that the state function itself has the meaning of an ‘*amplitude of probability*’, an unusual and ‘purely abstract’ entity.

The measurement of a quantum variable related to a physical system through an appropriate experimental arrangement yields the various eigenvalues corresponding to the system’s state function statistically, each with the probability for this state. Such a formulation avoids (and even rejects) the association of a definite physical quantity with a given system, except for quantities obeying a superselection condition (corresponding to one unique value determination): a quantum system can only have a *probability* for a given state, and a given quantity (though numerically valued) is not a *property* of a system but only the possible result of a measurement with the appropriate probability. Quantum variables are therefore not thought of as existing by themselves, but as conditional properties, depending on the observational arrangement: for that reason they have most usually been referred to as ‘observables’.

This way of formulating a physical theory was new in physics when it was proposed as a solution to the difficulty in interpreting the ‘wave’ function and the other quantum theoretical quantities (or variables) directly in terms of physical entities, a difficulty that had appeared in Schrödinger’s ‘wave equation’ and Heisenberg’s ‘transition matrices’. The dispersion in time of a wave along its path forbade consideration of the state function  $\psi$  of *wave mechanics*, describing a physical system also possessing particle-like properties, as the amplitude of a wave (or even the amplitude for a wave packet) (see Schrödinger (1926) and also various contributions in Bitbol and Darrigol (1993)). The same consideration applies to a system whose state function has the form of a linear superposition of ‘eigen’ or basis states: only one of these basic states or components of the superposition (statistically distributed) would correspond to the result of an experiment performed according to an appropriate preparation for a set of compatible variables, and be ‘physical’ at the same time. The function  $\psi$  was therefore to be understood as a ‘probability amplitude’, an abstract concept not considered

<sup>†</sup> After the first developments of ‘wave mechanics’ by Erwin Schrödinger (Schrödinger 1926) and of ‘matrix mechanics’ or quantum mechanics in the strict sense by Max Born, Werner Heisenberg, Pascual Jordan and Paul Dirac, which were proved to be equivalent as for their results (for historical descriptions, see Jammer (1966, 1974), Mehra and Rechenberg (1982–1987)), quantum mechanics was formulated in a more rigorous mathematical way, notably by Paul Dirac, Pascual Jordan, John von Neumann, David Hilbert and Hermann Weyl. See Dirac (1930), von Neumann (1932), Weyl (1932) and Pauli (1980).

<sup>‡</sup> It is often called a ‘wavefunction’. ‘State function’ (or ‘state vector’) is a more adequate expression for the representation of a system that is not reduced to a mere wave.

<sup>§</sup> The Hilbert space is built up by functions  $\psi$  whose modulus squared ( $|\psi|^2$ ) is integrable.

<sup>||</sup> Schrödinger’s equation in non-relativistic quantum mechanics and Dirac’s equation in the relativistic case.

truly physical. As for the operator-variables of *matrix* or *quantum mechanics*, they had been explicitly introduced to get amplitude transitions between atomic energy levels without worrying about non-observable (in principle) trajectories (Born and Heisenberg 1928; see also chapter 6 of Heisenberg 1969).

Other features of the quantum algorithm complete those already mentioned and we address a few of them. Only a (complete) set of ‘compatible’ variables (whose operators commute between themselves) can give the complete representation of the states of the system. Incompatible (or conjugate) sets of quantities (represented by non-commuting operators) lead to the Heisenberg type inequalities between the corresponding classical quantities (the so-called ‘uncertainty’ or ‘indeterminacy’ relations) and give rise to independent (‘complementary’) descriptions of the system, through independent projection bases for the state function. This property of quantum theory is sometimes referred to as ‘complementarity’, but this word is more adequate when used to qualify the classical corresponding representations, as we shall see, and the word ‘complementarity’ characterizes more adequately the philosophy of knowledge developed by Niels Bohr.

To sum up the status of the standard physical interpretation, wave mechanics proposed a *wave equation without real waves*, whereas quantum mechanics permitted calculation of *atomic transitions without a spatial representation* of these transitions (trajectories), and therefore the object of the quantum theoretical description was, so to speak, evasive (unless, as we shall see later on, one can imagine a kind of physical object different from the classical ones of wave or particle). In the usual conception of quantum mechanics, the wavefunction or the state function used as the description of a physical quantum state cannot be regarded as a direct representation of the state, and its relationship with the physical state is *only formally* analogous to that of a wave amplitude with its wave: a similar equation holds for both, but they possess different physical meanings, namely a ‘direct’ meaning for the second (classical) one and a quite indirect meaning for the first (quantum) one. This interpretation problem stands today, just as it did seventy years ago. Nevertheless the generalized use of quantum concepts through quantum theory has since that time led to some physical and conceptual clarifications that might help not to solve the problem given in these (restrictive) terms, but to formulate it in a different way and thus perhaps avoid and resolve the difficulty.

### **3. The difference between quantum mechanics and the classical theories: mathematical formalism and interpretation**

The difference between quantum mechanics and the previous physical theories has been frequently referred to stressing the particularly abstract and highly mathematical concepts of the first, as if the concept of the second were more ‘concrete’ or direct. Niels Bohr used to say that the mathematics used in classical physics reduced to real numbers, whereas quantum physics makes use in an essential way of the pure imaginary number  $i = \sqrt{-1}$ , which figures prominently in Schrödinger’s equations and, above all, in Heisenberg’s ‘uncertainty’ relations between conjugate quantities, as if ‘i’ were, so-to-speak, the mathematical counterpart of ‘complementarity’ (see Bohr 1958). Actually, it does not seem that the high degree of mathematical abstraction by itself makes the difference: when new concepts entered physics, it used to be in a mathematical form whose ‘direct’ character often looked problematic on first inspection, needing interpretation and starting philosophical debates (consider, for instance, continuous quantities as ‘simple’ as space and time coordinates when they took the form of differentials, from the end of the 17th century up to the mid-18th century).

The difference between the classical and the quantum domains of physics consists not so much in the *nature of the mathematics used* as in the *physical meaning of their use*: mathematical concepts and relations in classical physics referred ‘directly’ (whatever their degree of abstraction) to entities having a physical existence and physical properties, whereas

in the standard presentation of quantum mechanics they refer only indirectly to physics. One might propose that the physico-mathematical formalism had a useful (and even powerful) *drift effect* on quantum theory, but a high price had to be paid for this: it seemed necessary to abandon the hope of obtaining from quantum theory any direct intelligibility of the quantum physical phenomena and of the physical systems responsible for these. Intelligibility, it was said, was provided by the mathematical scheme plus the rules of physical interpretation, and would not deal with *physical existing quantities* but with *observational data* and ultimately with *observed quantities and measurement processes*.

At this stage, one may wonder whether one cannot dissolve this difference. There is, indeed, a general feeling among physicists that one should have a unique way of thinking in physics. *The passage from the classical to the quantum theory*, and vice-versa, considered in physics as well as in its different modes of representation, have remained from the beginning the most basic concern for the interpretation problems in quantum mechanics. Underlying this difference, which is effective at the physical conceptual level, is a questioning about the aim of physics and the nature of its object. Such questions have more than a flavour of epistemology and philosophy, and whether one accepts this or not, the 'new deal' for physical knowledge was actually left to philosophical choices. In the debate about the interpretation of quantum theory, positions on determinism, subjectivism, observationalism, empiricism, realism have continuously interfered with the discussions on conceptual and theoretical problems.

Those who accept this situation, according to Bohr's line and with some differences from Heisenberg and Born to Dirac, admit that the traditional notion of physical knowledge (which originated in classical physics) based on realism, objectivity and determinism had to be abandoned and replaced by a new one. In the new conception one must admit that the properties of a physical system cannot be considered independently of the conditions of its observation, which in turn necessarily brings us ultimately to macroscopic observation and measurement devices. For these are adapted to our senses, and relative to quantities of classical physics, that are supposed to be more natural than any other.

From this point of view, the traditional question concerning the nature of the system described by a state function is turned around, and importance is assigned, theoretically and conceptually, not to an indescribable hypothetical physical system considered by itself, but to the very description and its operational character. The state function would definitely not represent a system but rather the knowledge that one is able to get about it through interaction with the system or, in other words, the *catalogue of its observed properties*. (However, note at the same time how very difficult it is to avoid using the word *system*, i.e. *object*). This operational approach of the meaning of quantum mechanics and of its state function puts measurement, instead of physical existing (or 'real') systems, at the centre of the theoretical representation.

Those who, on the contrary, do not accept this 'interpretation' situation, replace the 'subjective' view by an objective one according to which the quantum theoretical representation refers to physical objects manifesting themselves in the phenomena and to which one should attribute properties. Their objection to quantum mechanics is both epistemological (as directed against the standard interpretation) and theoretical (quantum mechanics, in their view, does not fully describe its legitimate objects and should be completed in that sense). To strengthen their position, they would have to find a more satisfactory quantum theory: this is a more *deterministic* theory for those (who, following Louis de Broglie, David Bohm, Jean-Pierre Vigi er and others, invoke supplementary 'hidden' dynamical variables<sup>†</sup>), or a more *realistic* one for others (who claim, with Einstein, the need for a representation of real physical states having properties of their own, deeper than the *mechanistic concepts* on which, in his opinion, quantum mechanics was built (Einstein 1948, 1949, 1953, see Paty 1995, 1999)). However, none has so far been able to overcome the limitations of quantum theory as it stands.

<sup>†</sup> In 1926 Louis de Broglie proposed a nonlinear state equation with his 'double solution theory'. His simplified 'pilot wave' theory was independently re-discovered and perfected in 1952 by David Bohm in a way that is not contradictory with the results of standard quantum theory, as it includes non-locality. See Bohm (1980), Bell (1987).

Quantum physics therefore seems unable by itself to solve the problem of the physical meaning of its own theoretical quantities. One would have to invent and accept a radically new view about knowledge in general to give full legitimacy to the powerful quantum mechanical description. Such was the dogma stated by the ‘orthodox’ Copenhagen interpretation, and one may think that it was a philosophical response to a physical problem that brought the search for solutions to a premature halt (such as Einstein’s belief). We shall not enter this philosophical debate here, but will try to restrict ourselves to the consideration of the physical meaning of quantum theoretical quantities, calling on as few external considerations as possible.

Generally speaking, the various positions in the quantum debate can be displayed as possible responses to the question of the price which must be paid if one wants to maintain the intelligibility of quantum phenomena. It will depend on what one is ready to give up and what one is not. Taking quantum mechanics as it stands as a satisfactory theory<sup>†</sup>, an alternative to the Copenhagen philosophical response would be to ask: at which cost is it possible to maintain the requirement of a *direct theoretical description of physical systems* (a claim usually called ‘realism’)? It would obviously involve a change in our understanding of the physical concepts relative to a ‘direct theoretical description’. We shall include such a possibility among the existing positions of the interpretation problem, although it has seldom been considered explicitly. It is, altogether, often implicitly admitted by today’s physicists even when they pay lip service to the complementarity interpretation. To pass to the explicit formulation would require a new precise conceptual definition of a *physical quantity* and of a *quantum state*. These are generally conceived, by subjective as well as by objective approaches, as being identical to the quantities given by measurement.

#### 4. Measurement process and the classical to quantum barrier

The problem of measurement in quantum mechanics has generally been considered as determining all the other aspects of the ‘interpretation’. It has to do with the relationship between the quantum phenomena and the macroscopic, classical ones, and with the transfer of information between the two domains. This relationship and this transfer go in both ways: *from the classical to the quantum*, if we consider that observation with macroscopic instruments yields information in classical terms that has to be integrated into the knowledge of the quantum systems; *from the quantum to the classical*, if we prefer in physics to consider the structural aspects before the informational ones, and admit that classical systems and properties are derived in principle from the properties of their quantum constituents. One can indeed consider the ‘measurement problem’ from either point of view.

First, to go from the classical to the quantum, a proper notion or theory of quantum measurement would constitute a basis for a sound quantum theory. The strict Copenhagen notion proposes measurement by classical systems as reference and thus does not need a quantum theory of measurement. Measurement is the scheme from which knowledge is acquired, and quantum properties need to be defined and determined through classical concepts, i.e. those responses from macroscopic apparatus (taking into account the specific quantum-mechanical features).

As it was formulated in the set of quantum theoretical definitions and rules, the measurement process of quantum systems by classical apparatuses revealed a difficulty with causality. The state equation is causal with respect to the state function, for it describes the evolution in time of the latter when we consider it ‘in itself’, independently of any act of observation. However, according to the quantum rules, when the system is submitted to an observation or measurement process, its state function reduces from the coherent quantum superposition into one of its components, as appears to be the case in actual experiments. This reduction seems to be at random, for when the same experiment is performed a number of

<sup>†</sup> For nuances, see what we wrote at the beginning.

times under the same conditions on the same system, one recognizes that reduction has occurred to all the possible states, with a statistical law corresponding to the probabilities of the states.

In the orthodox interpretation, as we said, this was not a problem, but a mere question of definition: indeed, each observation or measurement redefines the state of the system. There is no need for a theory of measurement, because measurement participates in the definition of physical quantum systems, once one admits that there is definitely (as derived from quantum phenomena) no clear separation between the object and the subject. For Niels Bohr there was a barrier of perception between the quantum and the classical domain, which was due to measurement. It resulted from there, in his view, that the knowledge of quantum phenomena cannot grasp these directly as they are, but has always to refer to classical descriptions.

A variant of the 'subjective' view admits considering measurement processes as an interaction between the quantum system (whatever it is) and the classical apparatus (see, for example, von Neumann's theory of measurement (von Neumann 1932), and the theory of Daneri *et al* (1962)).

'Reduction' was a problem to those who were not content with the 'subjective' notion of the state function and wanted a description of the physical system itself. Such an instantaneous projection or 'collapse' to one of the possible states was difficult to admit because, precisely, it would be acausal. It has been the purpose of the attempts at an 'objective quantum theory of measurement' to give an account in purely causal terms of the transition. Objective notions require also in general a description of the interaction between the quantum system and quantum parts of the macroscopic measurement device, yielding finally the observed 'one basis state reductions' associated with the appropriate probabilities as frequencies of events<sup>†</sup>. Some objective notions consider 'no-reduction processes', where the observed reduction of the state function to one of its components on the prepared basis is not due to a physical interaction, but this component is just chosen. The reason or the modality of the choice differs with the various proposed theories: it may be a mere perspective effect, like in Hugh Everett III's 'relative state' theory of measurement, or in David Bohm's causal and non-local theory with hidden parameters<sup>‡</sup>.

Note that today physicists working on the developments of dynamical quantum theories consider the second point of view, i.e. of the passage from the quantum representation to the classical one, to be more fundamental than the first, in agreement with the most current theoretical problem for physics, namely the unifying programme for the elementary theory of matter. We shall stop here by noting the following: even without disposing of a quantum theory of measurement, the practical interpretation rule of the quantum algorithm that connects observed classical quantities with quantum ones is sufficient to yield, at the present stage, a representation of physical quantum systems, whatever their 'reality', on which one can rely and formulate such a programme.

If this is the case, one would admit implicitly that a 'quantum theory of measurement with the help of classical instruments' is not necessarily a requirement for physics to progress further. One would assume, at least provisionally, that the present quantum theory would not have to be changed if the representation of quantum states was a direct one and the quantum entities would describe objects existing in a quantum world with indirect access. This is not to deny the persistence of the problem of 'quantum measurement', but to relativize it and to lean more towards the epistemological interpretation than to the theoretical physical description. Viewed from both sides, the passage from one domain or representation to the other is a reduction, but of a different kind. Namely, reduction of (classically described) macroscopic matter to its (quantum) elementary constituents for the second, and reduction of the specific quantum features to classical (complementary) description for the first.

The notions or theories of quantum measurement, subjective as well as objective, have in common that they admit as a physical state only those 'basis states' that are given as a result

<sup>†</sup> The literature on this subject is enormous. A representative sample is given in Wheeler and Zurek (1983).

<sup>‡</sup> The original papers (published, respectively, in 1957 and 1952), have been reprinted in Wheeler and Zurek (1983) and Bohm (1980). See also Bell (1987).



of measuring quantities. These quantities have been chosen at a ‘*preparation*’ stage, defined by the type of experimental arrangement, and these data obtained are, by definition, numerical values. Take, for example, a particle of spin  $J = \frac{1}{2}\hbar$ , for which an apparatus is designed to measure the  $z$ -component of the spin. (This choice, allowed by the compatibility of both the quantities  $J$  and  $J_z$ , i.e. the commutation of the corresponding operators, precludes any simultaneous determination of the other components, since  $J_z$  and  $J_x$  or  $J_y$  do not commute and are therefore incompatible quantities).

A measurement of the component of the spin  $J_z$  with the help of a Stern–Gerlach magnetic apparatus will yield either  $J_z = +\frac{1}{2}\hbar$ , corresponding to a spin-up state, or  $J_z = -\frac{1}{2}\hbar$ , corresponding to a spin-down state. If the beam of particles is polarized with spin *up* (spin *down*), the measurement of it will yield  $J_z = +\frac{1}{2}\hbar$  ( $-\frac{1}{2}\hbar$ ) as the only result. If the beam is not polarized, measurement of it will give the value  $+\frac{1}{2}\hbar$  with 50% probability and the value  $-\frac{1}{2}\hbar$  with 50% probability. The state function before detection is a superposition  $\psi = (1/\sqrt{2})(\psi_{\text{up}} + \psi_{\text{down}})$ , but the measurement process reduces (or projects) it only on one of the states of the superposition: only the latter states are generally considered as being physical. The physical state before measurement is supposed to be unknown, determined only at the moment of measurement. For some (like Heisenberg), this determination occurs through the interaction of the superposed state with the measurement device, and there is no other way of speaking of a physical state. A system can be spoken of and described only when it is measured and not before.

One may ask how the connection between measurement and knowledge actually works. We have first to specify the physical meaning of the state function  $\psi$ . Within the theory, it represents the system under study. The state function is given a determined precise form and content from experiments measuring the relevant quantities ( $A$ ): from the numerical values (corresponding to the solutions of the eigenvalue equation ( $A\psi = a\psi$ ), associated with given frequencies  $c_i$  ( $= |\psi_i|^2$ ) (in statistical experiments), one gets the weighted (vectorial) sum of the corresponding states ( $\sum_i c_i \psi_i$ )<sup>†</sup>, that describes the state in the theory (the object state of the theory). Note that this state is an invariant in the following sense: it remains the same for another choice of basis states (defined by another set of commuting quantities); it is an invariant in the vectorial sense (in Hilbert space).

From the point of view of quantum theory dealing with quantum systems, this invariant or basis-independent character is important in that it frees the state from its contextual determinations. Some tension or duality of concern between the measurement approach and the theoretical approach can be diagnosed here, for one might ponder about the meaning of the reconstituted invariant state function in construction of the theory if it does not correspond to a physical state. The current quantum physics (of fundamental interactions) seems (fortunately) to have forgotten that, according to the standard interpretation, its favourite conceptual and theoretical tool has no meaning as a physical entity, but only its contextual projections have. One is therefore tempted to think that the standard interpretation of quantum mechanics has been restricted by limiting itself to the supposedly unavoidable transition from the classical to the quantum domains, when the fundamental issue is on the quantum level itself where theoretical physics is actually operating with a Hilbert vector entity, associating it with a theoretical physical content.

Clearly, the reconstitution of the state function from measurement is an *intellectual* procedure that is no more situated at the level of *perception* that had been claimed as referential for measurement processes. For theoretical knowledge does not stop at the level of perception but requires understanding, in order to formulate relations between concepts. In other words, our knowledge is not (and has never been, even in classical physics) restricted to our observations and measurements, but requires mental, symbolic, theoretical constructions.

We now turn away, for some time, from the measurement problem, and examine the physical meaning of the state function further.

<sup>†</sup> With an adequate normalization factor.

## 5. The superposition principle and quantum properties

When physicists speak today of a (quantum) ‘elementary particle’ (for example, a proton, or a quark), they implicitly mean that it is described by ‘quantum numbers’ that are ‘eigenvalues’ of the operators representing the adequate physical quantities. However, this representation is at the same time based fundamentally on the superposition principle, which is generally given in the epistemological debate on quantum physics as a purely mathematical and formal property. It is therefore of primary importance to understand just what the relation of this ‘formal’ principle to physical phenomena and systems is: whether it is purely formal and indirect or direct (albeit mathematical), that is to say entailing physical properties which can only be related to it.

Considering a physical system and its description by the state function, one attributes, so to speak, to the principle of superposition the responsibility of making any ‘direct physical interpretation’ of the state function in ‘classical terms’ impossible; however, one also gives it, from the point of view of the mathematical formalization, the credit for being responsible for *all the specific characteristics* of quantum systems. It is the most economical mathematical expression of these characteristics, a feature which, in theoretical physics, is generally attributed to a *physical principle*. The quantum debate nowadays can no longer avoid consideration of this problem, as many recent physical results have reinforced the *physical predictive power of the superposition principle*. We shall mention some of them, although rather briefly, as illustrations of the possibly increasingly *physical character of the state function*†.

Let us begin by noting, as all quantum concepts are parts of an overlapping whole, that the difficulty of considering a (quantum) physical ‘particle’ state as a linear superposition of its various possible (eigen or basis) states was mainly due to the connection of this property with non-locality. It is now admitted as a fact that non-locality is inherent in quantum systems (see further). Other direct consequences of the superposition principle have been shown to be fully physical, notwithstanding their initially apparently purely formal character inside quantum theory. By *physical* I mean the correspondence with *existing reported phenomena* at the quantum level, and which can also eventually manifest themselves at the macroscopic level.

*The indistinguishability of identical quantum systems* is one such property. The state function of a system constituted by identical quantum particles is constructed from those of the constituents ( $\psi_1$  and  $\psi_2$ ) through the superposition principle so as to obtain the quantum statistical behaviour: it must be symmetrical with respect to permutation for bosons ( $\psi_{12} = (1/\sqrt{2})(\psi_1 \otimes \psi_2 + \psi_2 \otimes \psi_1) = \psi_{21}$ ) and antisymmetrical ( $\psi_{12} = (1/\sqrt{2})(\psi_1 \otimes \psi_2 - \psi_2 \otimes \psi_1) = -\psi_{21}$ ) for fermions. These ‘formal conditions’, imposed by strong albeit limited original data (as recalled above: Planck’s radiation law, the electronic occupation of atomic levels or Pauli’s exclusion principle), entail enlarged powerful physical predictions, as consequences of the mathematical form of the state functions.

For a system of *identical bosons*, there is a possibility of accumulating an arbitrary number of them in the same state. Physical phenomena theoretically predicted as consequences of this simple or ‘economic’ theoretical feature have all been observed, and so are *optical pumping* and *the laser effect*, *superconductivity* (through Cooper’s electron pairs), *superfluidity* and, even more striking, *Bose–Einstein condensation*. There, tens of thousands of atoms of a given metal are accumulated in the ‘zero-point energy’ state, very close to absolute zero temperature, as recently observed (up to a macroscopic scale, the condensate instantaneously occupying all the space within its reach and climbing on the vessel’s walls) (Griffin *et al* 1995, Cornell and Wiemann 1998). For a system of *identical fermions*, the fact that a given state can be occupied by just one system at most has consequences not only at the atomic level (hence governing nothing less than all the *organization of matter*) but also at a highly macroscopic

† A more detailed analysis is proposed in a forthcoming publication (Paty 2000b).

one as well, such as the structure of *degenerate stars*: *white dwarfs* and *neutron stars* maintain their equilibrium between the gravitational tendency to collapse and the degeneracy pressure of electrons in the first case and of neutrons in the second (resulting from the impossibility for them to fall in the same state).

It is actually tempting to see in such a theoretical feature as *the indistinguishability of identical quantum particles*, which enables us to predict and understand such rich and uncommon physical phenomena that are otherwise inconceivable, the mark of a theoretical description of 'objects' by the state function which is more rigid than usual. This might appear paradoxical for a description that, on the contrary, has long been considered as lacking knowledge (beyond visualizability: recall the claims of subjectivism, of indeterminism, and of incompleteness as well). In the quantum domain, the *state function* of a physical system covers all the properties that can be attributed to that system.

Reverting to the usual way of thinking, we might say that, in this sense, quantum mechanics provides more knowledge about the described physical systems than classical physical theories used to do: in the latter *distinguishability* of identical particles or systems might be seen as nothing more than the expression of *our ignorance* about them (what allows one to distinguish them is not given by the theory). *In this respect* quantum theory provides a complete equivalence between the theoretical tool and the physical object, as no more information is contained in the object described than in its theoretical description (on the contrary, classical physics takes the objects it describes *as empirically given*, be they a falling stone or the Moon). This may seem paradoxical in terms of common sense, but the understanding of phenomena requires it (so it seems to me).

*Non-separability*, or to be more precise, *non-local separability*, is another characteristic of quantum systems which is directly connected with the theoretical properties of the state function and which has been shown to correspond to a factual property. Quantum (sub-) systems that are initially correlated in one state and are thereafter dissociated maintain the same correlation in future independently of their space separation. This feature of quantum theory had been pointed out by Einstein in his attempts to fix the description of quantum systems by their state function independently of measurement (the 'EPR argument') (Einstein 1935). Einstein felt the necessity to invoke a *principle of separability* guaranteeing the dynamical independence of spatially distant subsystems, knowing nevertheless that such a principle did not exist in quantum theory. He showed that, requiring this principle, quantum mechanics could not describe individual subsystems but only statistical ensembles<sup>†</sup>. Schrödinger spoke, for his part, of an 'entanglement' of quantum systems. Bohr objected that one cannot consider the systems independently of their measurement conditions and therefore the measurement of one of the EPR subsystems is also a measurement of the other. The correlation of the systems was in principle inseparable from that of the measurement apparatus: there was no point in speaking of the systems by themselves (Bohr 1935).

An important step was accomplished by Bell's theorem on locality that demonstrated an incompatibility between quantum mechanics and 'non-local hidden variables', extended afterward in a more general fashion to any kind of 'locality', corresponding to Einstein's 'principle of separability' (Bell 1964)<sup>‡</sup>. This incompatibility was later tested by correlation experiments on distant systems that concluded in favour of quantum mechanics (Freedman and Clauser 1972, Aspect 1983). Hence *non-local separability* of quantum systems was established as a *physical fact*. One must first observe, however, that it was present in the theoretical formalism, as the impossibility of factorizing the state functions of the subsystems, and to extract one of them from the overall system formed initially (unless one changes the initial conditions, and the state function of the overall system). Hence its theoretical meaning depended on the meaning one attributed to the state function, so that it has been understood in various manners.

<sup>†</sup> This conclusion comes out particularly clearly in Einstein (1948, 1949, 1953). See Paty (1995, 2000a).

<sup>‡</sup> This historical paper is reproduced in Bell (1987). Further results were obtained in particular by Greenberger *et al* (1989), and Mermin (1990). For more details, see, for example, d'Espagnat (1984, 1994) and Paty (1986, 2000b).

The evidence for quantum correlations has often been considered as a victory of Bohr's interpretation of quantum mechanics (an operationalist view) over Einstein's 'realistic' demand. It was certainly *a victory of quantum mechanics over a principle of separability external to it*. However, quantum mechanics is a physical theory and should not be identified with Bohr's interpretation or philosophy. And there is implied, in the correlations between systems, more than there was in Bohr's response, which would not have been able, by itself, to specify the quantum correlations through which non-local separability revealed itself as a *property of quantum systems* (Paty 1995, 2000a).

From the objectivist and realist point of view, a variety of conclusions have been given about the meaning of the quantum correlation (apart from pure refusal or doubt about the validity of Bell's theorem, or of the results of the correlation experiments). For some people who take localization in space as a requisite for physics, this property of quantum systems is a dynamical one, which reintroduces some kind of instantaneous action at a distance of the Newtonian type, one being left with a problem of compatibility with special relativity. In such a case the theoretical description by the state function as it stands would be insufficient.

However, there is, from an objectivist or realist point of view, another, simpler and direct way to live with this property and consider at the same time that the state function theoretically represents a physical system. Admitting non-local separability as a basic *fact* of quantum physics (in the same sense as the relativity principle, for instance, is a basic fact for classical field theories), quantum theory appears, precisely, perfectly adequate to give account of it: there is no need to require any additional property (such as a principle of separability or locality) taken from outside of quantum theory. Since quantum systems are defined by quantum theory, their attributable properties are covered by their state function; they are naturally extended in space and entangled with the other subsystems co-native to them (and special relativity stays outside of this, as quantum systems are not material points in space but extended systems) (Paty 1986). Again, the state function is sufficient by itself to represent and describe the fundamental property under consideration.

## 6. Probabilities and the single event description problem

The state function is an amplitude of probability; with respect to observed or measured quantities, its modulus squared gives the frequencies of the possible (or eigen-) values for the corresponding eigenstates. In the context of the standard interpretation referring to observation, probability and statistics were usually identified<sup>†</sup>. Actually one can recognize a difference between them, from the point of view of their functions: probability (derived from a 'probability amplitude', which is a physical concept, not a mathematical one) has its meaning inside physical theory, while statistics refer to experiments and measurements.

This conceptual difference might be useful in clarifying the status of the description of individual systems in quantum physics, a question that has been considered unimportant in the standard interpretation insofar as counting quantum particles was considered as requiring a measurement and therefore perturbing the state (and finally one could never ascertain whether one deals with only one particle). For example, interference patterns produced by having quantum particles interfere in a two-slit interferometer are always obtained statistically, although the process is attributed, from the behaviour of the state function, to a self-interference of quantum particles (Dirac 1930). (Recall that it is this simple quantum phenomenon that had led Born to his probabilistic interpretation.) The statistical meaning of the state function permitted, in the Bohrian perspective of complementarity, to reconcile the wave and particle aspects of quantum phenomena. If these dual aspects are incompatible for individual (single) systems, they can be assumed for an ensemble of systems. One may ask, however, what would happen if there were no need for a complementary description, which would happen if

<sup>†</sup> See, for example, the views shared on this point by both Einstein and Born, the inventor of the 'probabilistic' (or equivalently 'statistical') interpretation of the state function (Einstein and Born 1969).

individual phenomena could be considered independently, i.e. without reduction to classical features.

This has indeed been possible, for about two decades, thanks to the production of extremely rarefied beams of quantum particles (photons, electrons, neutrons, atoms, etc) with a high time resolution (see, in particular, Pflugor and Mandel 1967, Grangier 1986). One knows that particles have crossed the interferometer one by one, without counting them on their path. Instead of doing one experiment with many (say  $N$ ) quantum particles, one is now able to perform  $N$  independent experiments with a single quantum particle (identical in all experiments). One obtains, at the end, the same interference pattern, with the difference that one is sure that every individual system has equally contributed to form the final pattern. If the impact on the screen appears for each one at random, the accumulated impacts reproduce the statistical distribution law.

Each phenomenon related to an individual system is a *quantum phenomenon*, collected on the screen through a *classical measurement* process (the impact of a quantum particle on a grain of the sensitive material on the screen). One is left with the usual problem of the quantum measurement process: identical quantum systems provide, after detection, different results, but endowed with probabilities corresponding to the amplitude of probability of their state of superposition.

The observed *diffraction phenomenon of single quantum particles* also yields probabilities having a meaning for individual events. On the whole, this phenomenon provides further evidence for a direct physical meaning of the probability amplitude, i.e. of the state function.

## 7. Coherent superposition states observed to propagate must be physical

When physicists consider a physical system as represented by a state function, they do not consider the latter as a pure mathematical entity or a catalogue of information. They treat it as if it really meant a propagating physical system. They think physically and only afterwards do they eventually interpret in the standard 'authorized' way. In particular, they treat as physical a state whose state function is a coherent superposition of basis states related to a given 'preparation' (or choice of compatible quantities represented by commuting operators). As a whole, this state vector is invariant under a change of the basis corresponding to a preparation with another (incompatible) set of quantities. Being a mathematical invariant, this state vector (or state function) is theoretically meaningful, much more so than its projected components.

It has been claimed in the standard interpretation, however, that a coherent superposition has no direct physical meaning, as it does not correspond to direct possible observation. By definition and construction, direct observation can only detect one of the basis states from a given numerical value for each relevant variable. This doctrine would, therefore, deny the name of physical state for a state of superposition, even if there were experimental evidence for it, and experimental evidence can also be obtained from indirect observation. Now, such evidence has recently been reported as 'decoherence experiments'. A two-state Rydberg atom (superposition quantum state) coupled with a double-valued electromagnetic field with very few photons ('mesoscopic' state) realizing a kind of 'Schrödinger's cat experiment' (Schrödinger 1935) has been detected as a superposition state in its propagation inside the experimental arrangement, for a small finite time, before its decoupling (or decoherence, within the macroscopic environment of the measurement apparatus (Haroche *et al* 1997)). This experiment can be considered as evidence for the physical propagation of a coherent superposition state. It gives other indications on the 'measurement process' and 'reduction of the state function': decoherence apparently arises from a thermodynamic process, which we shall not discuss here (see Zurek 1991, Omnès 1994).

We content ourselves here with the new fact of a visualization of a state of coherent superposition propagating through space, and note that superposition states observed to propagate must be physical.

## 8. Extension of the meaning for quantities: beyond the merely numerically valued

From the preceding argument, the description of a quantum system by the state function of quantum theory appears in many respects to be as powerful in predicting as in explaining physical phenomena in the quantum domain. Everything in our understanding would suggest considering this mode of description as a *physically direct* one: the state function as it stands represents a *physical system*, and this has *dynamical properties* given by the quantum variables as expressed by the formalism. The only restriction to that ‘simple’ view is that this theoretical concept does not have the meaning of *simple numerically valued quantities* as one is used to in ‘pre-quantum’ physics. However, we know that the quantum domain of phenomena exceeds the classical one and that the understanding of the former must be different in some way from the latter. In contrast to the proposed view, the standard interpretation of quantum mechanics considered as untouchable the classical understanding of physical concepts that they referred to a primacy of measurement procedures, and preferred to give up the idea of ‘real physical systems’ having properties.

Understanding, as we practice it in quantum physics, cannot be reduced to perception, nor physical description to observation. We may choose another way that would provide the quantum concepts with a direct physical meaning related to the main function of physical quantities which is to express physical relations between them, and consider their connection to observation and measurement as secondary and indirect. For this, we would have simply to extend the usual concepts of *physical state* and *physical quantity* of a system, which have been up to now restricted to definite *numerical* values, so as to include the *mathematical forms met within quantum physics*, i.e. state vector superpositions, matrix operators. This means recognizing that the understanding of the quantum domain is given through the quantum theory and that physical thought has been driven by the formalism of quantum theory in a more powerful way than was initially thought. It can be stated also that the power of quantum physics comes from its mathematical expression and not from its ‘interpretation’ in terms of knowledge of the natural world in general.

The intuition of some of the founders of quantum theory had led them to think in such a way, up to the point where they considered the connection of the quantum to classical, that they gave way to the then current observationalist view. Consider Dirac’s notion of *q*-numbers as an extension of the usual *c*-numbers (see Darrigol 1992), or his conception of theoretical research in physics, as he formulated it when he was discovering the concept of antiparticle to which the mathematical form of his equation of electron motion was leading: the ‘most powerful method of advance that can be suggested at present is to employ all the resources of pure mathematics in attempts to perfect and generalize the mathematical formalism that forms the existing basis of theoretical physics, and *after* each success in this direction, to try to interpret the new mathematical features in terms of physical entities. . .’ (Dirac 1931, with emphasis added by MP).

The conclusion or the moral of our story about the direct physical meaning of the fundamental quantum concepts of state function and quantity is that the clue is contained in the mathematical formalism. However, this was not obvious from the start, for it is not sufficient to make use of mathematical relationships that are convenient for the solution of a physical problem to be assured that the elements of these relations can be identified with physical entities. However, one may consider that today quantum phenomena as displayed above justify a simpler and more coherent representation in such terms.

Thus a solution of the ‘unsolved problem’ of the physical meaning of quantum concepts and quantities seems to emerge, if its consistence is fully confirmed. This solution would be more an example for a historical situation where a problem considered as having no solution in the existing narrow frame of thought can actually get one by simply widening the meaning of its concepts. Such a situation has been quite frequent in the history of mathematics (think only of the various extensions of the concept of number, which were the only way to get rid

of the mysteries of the irrational and later on of the imaginary quantities), and also occurred in theoretical and mathematical physics as well.

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