REMARKS ABOUT A "GENERAL SCIENCE OF REASONING"

Comments on Peter Clarke's « Frege, neo-logicism and applied mathematics ».

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1.

As I am not at all a specialist of Gottlob Frege's work, my comments intended initially to be focused on an aspect that emerges from the last part of Peter Clarks's paper « Frege, neo-logicism and applied mathematics »[1]1, where he treats the question of « applied mathematics », an aspect that appealed to me and that was triggered by Frege's relationship between numbers and concepts, and reasoning. Starting with this concern, I have been led by my subject to propose some considerations about foundations and rationality which will go – briefly - in two directions. The first direction is that of a distinction between logical and rational foundations, whilst the second direction is that of taking into account as a fact the historical development of mathematics among the sciences, which modifies the terms of any foundational program. In delineating these considerations, I found that what I had in mind could apply to mathematics itself as well as to « applied mathematics », thus deviating somewhat from my first explicit purpose. In conclusion I shall consider the possibility of a rational foundation programme for mathematical and physico-mathematical sciences which would take into account the changes in the scientific contents and the widenings of the forms of rationality that, in my view, make these changes possible. Such foundations for knowledge would not be any more static, but dynamical and would be possibly considered only retrospectively : they would be « forward foundations », in a sense that will be discussed in detail elsewhere[2].

F. Stadler (ed.), *Induction and deduction in the sciences*,
Kluwer, Dordrecht, 2004, p.185-193.

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[1] Peter Clarke, « Frege, neo-logicism and applied mathematics », in this volume.

[2] Michel Paty, « Des fondements vers l'avant. Sur la rationalité des mathématiques et des sciences formalisées », Contribution to the Colloque International «Aperçus philosophiques en logique et en mathématiques. Histoire et actualité des théories sémantiques et syntaxiques alternatives», Nancy, 30 sept.-4 oct. 2002 (forthcoming).

2.

Let us first start from Frege's concerns with his logicist programme for mathematics and for science3[3]. According to him, the logical foundation of arithmetic (if it were possible, which he hoped) would entail that arithmetic can be seen as akin to a general science of reasoning about objects. This is made evident by Peter Clark's statement : « To the question "Why does arithmetic apply to reality ?", the logicist provides the clear answer : "because it applies to everything that can be thought". It is the most general science possible. » And Peter Clarks emphasizes : « In the simplest case for which the question arises – the application of cardinal numbers – [Frege's] solution is that arithmetic is applicable to reality because the concepts, under which things fall, themselves fall under numerical concepts. »[4]

A question arises at this stage : is not this strong connexion - a quasi identification – between concept and number, related to the fact that Frege's conception of knowledge is, as far as I know, a pre-kantian realism, according to which objects that are described by thought correspond directly to objects that exist in reality ? And objects that exist can be counted, this being their first qualification. Thought would begin by counting, and it would be possible for thought to construct everything from numbers.

To this rather simple view, one can object that the entities that are described by scientific thought (mathematics, physics, etc.) are never known directly, but through our mental and conceptual elaborations : the concepts by which we represent them are not only numerical ones. They are symbolic representations, and not all symbols are numbers. Relations of symbols can make sense without being relations of numbers (such are, for example, letters of the alphabet clustered in words)[5]. Numbers, as such, are too restricted to be able to represent any kind of possible objects. Even if we restrict ourselves to mathematical – and « applied mathematical » - knowledge we see that other types of mathematical entities can be invoked to express concepts which are of the quantity type and of the continuous quantity type, for example geometric or algebraic, including differential ones for continuity and change. Moreover, these quantities are not only related to « measure » (in the meaning of the word in XVIIth century, i.e, proportions, ratios), but also to respective positions (as Descartes said, « order », in his 1627 *Rules for the direction of the mind*) ; or, as Riemann wrote in his 1854 *Dissertation on the foundations of geometry*, speaking on the properties of manyfolds, they are not concerned only by metrics, but also by topology (*analysis situs*).

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3 Gottlob Frege, *Die Grundlagen der Arithmetik. Eine logisch-mathematische Untersuchung über den Begriff der Zahl*, Breslau, W. Koebner, 1884 (Engl. transl. by J.L. Austin, *The Foundations of Arithmetic*, Oxford, Blackwell, 1950. French transl. by C. Imbert, *Les Fondements de l'arithmétique*, Paris, Seuil, 1970) ; Gottlob Frege, *Ecrits logiques et philosophiques*, transl. in French from German, Paris, Seuil, 197 ; J. van Heijenoort, *From Frege to Gödel. A source Book in Mathematical Logic (1879-1931)*, Cambridge (Mass.), Harvard University Press, 1967.

4 Peter Clarke, *ibid.*

5 It is true that Frege himself considered a more general symbolic notation than pure numbers (ideography).

As to the applicability of mathematics, it is directly connected with the various possible types of magnitudes or quantities which can be used in the « application » field (for instance, physics). Take, as an example, geometrical reasoning, and, in particular, the qualitative study of curves represented by systems of differential equations as inaugurated by Poincaré at the end of XIX th century. Exact solutions, with numbers, are overrunned by qualitative ones, which reasoning can grasp much more powerfully. This shows how mathematical reasoning (and by way of consequence, reasoning in other scientific disciplines where mathematical thought is used) is not restricted to numbers. To these considerations, on can add quantum theory, be it quantum mechanics or quantum field theories, and the mathematics that goes with it, i.e. state vectors of Hilbert space and linear operators acting on them with their non commuting properties. It is difficult to reduce such conceptual entities merely to numbers. They actually are considered in their complex mathematical form, which entails their relational capacities, by mathematicians or theoretical physicists (and even by experimental physicists[6]) when they think about them and operate with them. And the common remark that « everything ends into numbers » if we evoke, for instance, numerical approximations for the solutions of equations, or for the comparisons with experimental data, or computer binary calculations for simulating problems, is not a satisfactory answer to the question of the intelligibility of such mathematical or mathematized entities, because it is a practical and not a principle and foundational one.

The impossibility to found mathematics, and even arithmetic, on pure logic, is at least coherent with such a state of things whose consciousness comes to us from the lessons of mathematical and physico-mathematical reasoning practice.

3.

After this preliminary comment let us come to the logic-versus-reason consideration. The basic question about Frege's program seems to me to be the following : is reason (generally speaking, as the mental function for understanding) to be fundamentally identified with logic ? (and consequently, to arithmetic ?) As we know, the answer is : no ; because already with mathematics we cannot identify mathematical reasoning with arithmetic alone (or refer to it alone) as we have argued, and even less with logic alone, as Gödel's theorem demonstrates.

The question can also be considered from the point of view of the intellectual activity in the mathematical field for which many mathematicians see a kind of duality and competition between logic and intuition, the first corresponding to the requirement of rigor, the second to the process (and necessity) of invention. Mathematical theories are theories about some mathematical content, and this mathematical content has not been given by purely logical operations of the thought. It implies some intuition, for its invention as well as for its understanding : intuition

[6] Michel Paty, « The concept of quantum state : new views on old phenomena », in A. Ashtekar, R.S.Cohen, D. Howard, J. Renn, S. Sarkar & A. Shimony (eds.), *Revisiting the Foundations of Relativistic Physics : Festschrift in Honor of John Stachel*, « Boston Studies in the Philosophy and History of Science », Dordrecht, Kluwer, forthcoming.

is a word – indeed, a concept – that many mathematicians and theoretical physicists, and also philosophers, use since the beginnings of philosophy, with a meaning that is not fixed from one to another and through the times. This is one of the reasons why the logicists would like to evacuate it from the language of metamathematics, and most contemporary philosophers from the language of the philosophy of knowledge. But its permanent use through the times – even with varied meanings, from Aristotle to Descartes, Kant, Poincaré, Hilbert, Weyl, Einstein -, testimonies the necessity of having a philosophical concept to maintain an aspect of knowledge, and of even the most precise and exact knowledge, which escapes mere reduction to logic.

Poincaré's statements about the opposition between the two mathematical dispositions of logic and intuition are well known : « Logic is for demonstration, intuition for invention »[7]. To Poincaré, a geometer who would lack intuiton would be in the same sitation as a grammarian who, although he would know all of grammar, would lack ideas. Hilbert himself expressed some similar ideas, which appears interesting and significant as coming from the pioneer of the axiomatization of geometry. He stated, in 1922, with a reference to Kant, that the sound matter of mathematics is given independently of any logic and that consequently mathematics will never be founded by logic alone, evidencing henceforth the reason of the failure of Frege and Dedekind (this being stated ten years before Gödel's theorem). To him, Hilbert, on the contrary, « the beforehand condition for applying logical reasonings is the presence of something given in the representation, some extralogical objects which intuitively happen to be there as an immediate experience, previous to any thought. (…) If the logical deduction is to be made firm, it has to bear on objects which can be grasped by all their sides, and which are such that their distinctive signs, their reciprocal relationships be identifiable with them, as something which is not reducible to anything else and which does not need it. »[8]

The non reducibility of the mathematical content (i.e. what is given in the mathematical relationships) to pure logic entails, in my view, that intuition in the sense of Poincaré and Hilbert pertains to reasoning : in no case it is erased under psychological considerations. For these mathematicians (as for the physicist Einstein), intuition is a kind of synthetic apprehension of the given by the understanding, this apprehension fully pertaining to what we call reasoning, which refers to « rationality ».

In a way, that one has to hold on both logic and intuition in mathematical reasoning, despite the opposition of these two functions of the mind, points at the fact that a structured content of knowledge cannot be rendered by its structure alone. The theoretical and conceptual content of mathematics (or of any rational knowledge) cannot be generated by its logical structuration. Actually, it is something more of the contrary : although we can recognize a posteriori that the structure is co-generated with the content, in our knowledge the structure comes

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[7] Henri Poincaré, « La logique et l'intuition dans la science mathématique et dans l'enseignement », *L'Enseignement mathématique*, 1, 1889, 157-162 (Repr. in Henri Poincaré, *Œuvres*, Paris, Gauthier-Villars (11 vols., 1913-1965), vol. 11, pp. 129-133.

[8] David Hilbert, « Neubegründung der Mathematik » (1922), in David Hilbert, *Gessamelte Abhandlungen*, Berlin, B. 3, 1935.

posterior to the mathematical content. To us, the logical structure comes out from the content once the latter is produced and made known to us : the (mathematical) content shows itself endowed with a logical structuration. When dealing with knowledge, we must consider the effective science which is produced and made intelligible, and when considering it, one cannot escape the fact of mathematical or scientific invention or creation. Even if mathematical reality (taken as all mathematical contents taken together) or physical reality (that one to which physics is refered, in the « real, external, world ») are considered separately from human mind and objectivated, as knowledge they are the product of intellectual mind activity. Any « foundational program », and the logical one itself, is a program about a given knowledge and cannot ignore these dimensions of knowledge. The invention process from which the (mathematical or else) knowledge content has been issued precedes the recognition of its logical structuration. One cannot invent, when reasoning, with logic alone, even if logic is implicitly present in this reasoning. For reasoning operates with objects, statements, symbols that are not univoquely given, neither at the start nor in the course of its operation. The mind (its function of understanding) has to choose and eventually to connect between them entities (propositions) that were previously unconnected. The mathematical philosopher Jean Cavaillès[9] stated something of this kind in his 1938 Doctoral dissertation, *Méthode axiomatique et formalisme. Essai sur le problème du fondement des mathématiques* : « Mathematics is richer than logic, insofar as it is an effective thought and any effective thought supposes applying abstract thought to an intuition »[10]. « To reject or to found a theory », he wrote in his *Remarques sur la formation de la théorie abstraite des ensembles* (his complemetary Doctoral dissertation), « is neither definitive nor devoid of degrees ; (…) it cannot be done simply by logical investigation ; (…) the pragmatic considerations of the militant mathematician tell the last word »[11]. As for him, according to another mathematical philosopher, by the way a Cavaillès' disciple, Jean-Toussaint Desanti, « he wanted to catch the "mathematical experience", in its necessary mouvement of constitution, and in it, consubstantial to this necessity, logic itself »[12]. And Gilles-G. Granger (also a former Cavaillès' disciple) emphasizes that for Cavaillès, the historical development of mathematics is rational and there is an objectivity of the mathematical becoming, while noting the fascinating paradox of the history of mathematics, which is at the same time imprevisible and rational.[13] Cavaillès himself, who had a spinozian inspired conception of an immanent development of mathematics, stated in his posthumous book *Sur la logique et la théorie de la*

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[9] Jean Cavaillès (1903-1944), prematurely carried off by death in the Nazis' hands in occupied France, where he had a leading Resistance activity, was a philosopher and logician.

[10] Jean Cavaillès, *Méthode axiomatique et formalisme, Essai sur le problème du fondement des mathématiques* (Thesis, 1937, 1st ed, 1938), Introduction by Jean-Toussaint Desanti, Preface by Henri Cartan, Paris, Hermann, 1981. (My emphasis, MP).

[11] Jean Cavaillès, *Remarques sur la formation de la théorie abstraite des ensembles* (Complementary Thesis, 1937, 1st ed, 1938), in Jean Cavaillès, *Philosophie mathématique*, Preface by Raymond Aron, Introduction by Roger Martin, Paris, Hermann, 1962, pp. 23-174.

[12] Jean-Toussaint Desanti, « Souvenir de Jean Cavaillès », in Jean Cavaillès, *Méthode axiomatique et formalisme. Essai sur le problème du fondement des mathématiques* (1981 ed.). Introduction by Jean-Toussaint Desanti, Preface by Henri Cartan, Paris, Hermann, 1981.

[13] Gilles-Gaston Granger, *Science, langage, philosophie*, Collection « Penser avec les sciences », EDP-Sciences, Paris, 2003, chapter on « Jean Cavaillès et l'histoire », pp. 76-84.

science, that « the true meaning of a theory does not stand in an aspect that is understood by the scientist himself as essentially provisional, but in a conceptual becoming which cannot be stopped »[14].

4.

We could transform Frege's worry about identifying reasoning with dealing-with-numbers into the following one : does reasoning in general have a direct connection with mathematical reasoning ? We consider the question, this time, taking mathematics as any kind (and all the kinds) of mathematical theories and concepts, those which are known to us, but also taking into account the fact, tought to us by the effectiveness of mathematics, as Cavaillès recalled, that mathematics are not closed and that more mathematics are to be invented. We are tempted to say, pursuing the parallel : more mathematics through more reasonings, more rationality modes, which are to be invented with them. For reasoning, and rationality, considered in general is (possibly) akin to mathematical reasoning, which includes the extension (by the creativity of the mind) of mathematics (inasmuch as rationality, in its exercising, implies the extension of what we are used to call rationality, or modes of rationality).

It is clear, from all what precedes, and from many other considerations which we cannot exhaust in this short space, that reasoning, either in mathematics or generally speaking, is definitely not to be identified with logic because logic is too restrictive and cannot by its only exercise, when one is reasoning about something, modify or generate the premises, for example when building theories (mathematical or physical ones, or other kinds of theories).

That reasoning is not reduced to logic, or even is not based on logic, does not mean, clearly, that elaborating scientific theories would be illogical for that. Logic is still present, be it implicitly, as a kind of regulation, a criterion along the path of the « working thought »[15] which makes emerge knowledges and forms of knowledge. Actually, we are no more, with this kind of considerations, in the domain of foundations of knowledge in the usual sense that considers foundations that would be already given, but in the domain of the extension (or growing) of knowledge. A question arises whether one can speak again of « foundations » when one knows that sound knowledge is subject to modifications, and even more, that it would not be knowledge in the full sense if it was to stay static. Can we consider with some meaning a foundation for a knowledge which has, as a fundamental property, to be dynamical, i.e. to be modified with time ? If this could be considered, it would have to be on the condition of thinking anew the concept of foundation. The property of stability is generally required for foundations, and we know that mathematical and more generally scientific knowledges are relatively stable, inasmuch as they are growing and in a way cumulating. New knowledge

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[14] Jean Cavaillès, *Sur la logique et la théorie de la science* (written in 1942, 1st ed., 1946), 3rd ed., Paris, Vrin, 1976.

[15] I mean, by the expression « working thought » what I would call in French : « la pensée au travail ». The idea of scientific or rational thought as a working action has been developed by Gilles-Gaston Granger, notably in his *Essai d'une philosophie du style*, Paris, Armand Colin, 1968 ; reed., Paris, Odile Jacob, 1988.

increases the precedent one, and its discovery and formulation base themselves on the previous one, whilst at the same time it can entail a complete change of meaning of concepts, theories and bodies of knowledge.

In short, stability has to cope with instability. Is this foundation ? We shall not solve the problem here, but only note that the exercise of reason is in all cases the only instance of judgement : reason and its operating forms of rationality, mathematical, physical, etc., and common ones as well. These forms themselves are changing, actually they are enlarging, as we suggested, to make the new knowledges possible and give us the intelligibility of these, and to continue understanding the previous ones, but in a somewhat modified way, from another point of view (generally, a more unifying one). The forms of rationality are changing, but their function stands and, with the function, something stable through its own motion and modifications. Such a problem can well be considered as a problem of foundation, not for logic but for reason. In renewed kantian terms : what are the conditions in the rational structures of the mind for a dynamical and at the same time relatively secure knowledge be possible ?

5.

To complete what has been sketched above, let us consider the question of the « application of mathematics » in its relation with the foundational problem and with the problem of the generalization of aspects of mathematical reason to reason in general. To say something of it in a few words, the so-called « application of mathematics », let us say in physics, is pulled by the external world which manifests itself empirically. But, at the same time, this « application » (or, to say it better, this « use of a mathematical tool for reasoning ») transforms the empirical, by assimilating it, into rational constructions and representations[16]. The legitimation of this transformation of an hypothetical-empirical into a rational-intelligible is given by its success in representing the physical phenomena, but also in anticipating them, and a condition for it, from the theoretical point of view, is the systematical character, which requires that the representation constitutes a theoretical system. (Examples are wellknown : classical mechanics, relativistic electrodynamics, general relativity, thermodynamics, and we can add quantum physics[17]).

The justification of this process of rationalization does not therefore come from beneath (from static foundations), neither does it come from before (as if all present and future knowledge was already contained in its previous forms), but it comes from forward. I mean that we know the justification and foundation (as

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[16] Michel Paty, « Intelligibilité et historicité (Science, rationalité, histoire) », in J. J. Saldaña (ed.), Science and Cultural Diversity. Filling a Gap in the History of Science, Cadernos de Quipu 5, México, 2001, pp. 59-95 ; « Les concepts de la physique : contenus rationnels et constructions dans l'histoire », Principia (Florianopolis, Br), 5, n°1-2, junho-dezembro 2001, 209- 240 (English version : « The concepts of physics : rational contents and constructions in history », in J. Margolis and T. Rockmore (eds.), forthcoming.

[17] For the last one, see in particular : Michel Paty, « La physique quantique ou l'entraînement de la forme mathématique sur la pensée physique », in C. Mataix y A. Rivadulla (eds.), *Física cuantica y realidad. Quantum physics and reality*, Madrid, Editorial Complutense, 2002, pp. 97-134 ; « The concept of quantum state : new views on old phenomena », *op. cit.*

firm ground) of a given piece of theoretical knowledge only when it has been already achieved or completed. Maybe this situation is also that one of nonempirical knowledge such as mathematics, as its seem to appear from what we have said previously. It seems indeed rather reasonable to think that if knowledge is taken as a dynamical process, and is unachieved, unclosed, its true foundation has not been already attained, and is not yet attainable with the means that we presently dispose of.

A last remark which would need larger developments, about formal considerations linked to the sciences of nature, which may clarify a legitimate use of the expression « rational reconstruction ». This expression, used for instance by Imre Lakatos, Elie Zahar and others, has the inconvenience to lead to think that construction of science by itself would not be rational (indeed, Reichenbach, Popper and many others thought so) for it needs a « rational reconstruction », when we have suggested, on the contrary, that even intuition in creative scientific work sits on the side of the rational. From what has been said about « mathematical application » to other sciences - and essentially physics is concerned -, we can then understand the meaning of a kind of « rational reconstruction » in the same way as we understand the meaning of « axiomatizing a theory ». Both are meaningful, not because there would have been a lack of rationality in the creation and in the discovery of new knowledge, or in a non axiomatized theory. (Rational) reconstruction and axiomatization actually consider the given theory from another point of view, the point of view of the « economy » of the propositions and of the theoretical structure. In these, the theoretical structure is rearranged in such a way as to show a deductive and logical sequence from transformed premises or starting principles or axioms. These premises, principles, axioms, carry the physical content (in the case of this science) which can be evaluated, and can eventually be considered henceforth as the new basic (foundational) and « natural » (or « reasonable ») concepts. The correlative obtention of a direct rational-and-logical view can be seen, as for itself, as a change – indeed, an extension – of the forms of rationality operating on this body of knowledge.

The sense of these reflections is that, despite the failure of the foundational program in the logical sense, there remains a meaning for a « foundational » concern, which is grounded in the function of rationality, that makes mathematical and scientific knowledge not only something known to us, but deeply intelligible to us.

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